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Small E_8 Instantons and Tensionless Non Critical Strings

Ori J. Ganor

origa@puhep1.princeton.edu
Department of Physics, Jadwin Hall, Princeton University
Princeton, NJ 08544, U. S. A.

and

Amihay Hanany

hanany@sns.ias.edu
School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, U. S. A.

Abstract

T-duality is used to extract information on an instanton of zero size in the $E_8 \times E_8$ heterotic string. We discuss the possibility of the appearance of a tensionless anti-self-dual non-critical string through an implementation of the mechanism suggested by Strominger of two coincident 5-branes. It is argued that when an instanton shrinks to zero size a tensionless non-critical string appears at the core of the instanton. It is further conjectured that appearance of tensionless strings in the spectrum leads to new phase transitions in six dimensions in much the same way as massless particles do in four dimensions.

1 Introduction

Recently Witten [1] showed what happens when instantons on the $SO(32)$ heterotic string shrink to zero size. The low energy six dimensional effective theory has an extra $Sp(1)$ gauge symmetry which is supported at the core of the instanton. The finite size instanton is obtained from the zero size instanton through a Higgs mechanism. In six dimensions there is no vector multiplet moduli space and therefore the Higgs mechanism is presumed to be the only relevant low energy dynamics.

In this paper we will explore some features in the problem of small instantons on the $E_8 \otimes E_8$ heterotic strings. A straight forward generalization of Witten's construction does not seem to work because the dimensions of the E_8 representations are too big. It was suggested by Witten [2] that the solution involves a new type of physics unknown before.

In [3] Duff and Lu constructed a supersymmetric solitonic one-brane solution of $N = 2$ $D = 6$ supergravity which is self-dual. This soliton was realized in [4] as a self-dual three brane of the type IIB string compactified on K3 wrapped on a self-dual two-cycle of the K3. When the K3 degenerates in such a way that the pullback of the complexified Kähler form on the two cycle is very small the tension of the one-brane (which is proportional to the area of the two cycle) is much smaller than any other scale in the theory. Another realization of this one-brane was discussed in [5]. A membrane stretched between two five-branes in the M-theory has the one-brane solution at the boundaries of the membrane. When the two five-branes coincide the tension of the one-brane vanishes. The constructions of [5] and of [4] are related as explained in [6].

Returning to low energy in six dimensions we see that a new type of dynamics may appear through the emergence of a tensionless self-dual string.

We asked ourselves the following questions:

- What can we learn about the $E_8 \otimes E_8$ small instanton from T-duality between $SO(32)$ and $E_8 \otimes E_8$?
- Is there any relation between tensionless strings in 6D and an $E_8 \otimes E_8$ small instanton?
- What happens when a 5-brane of the M-theory on $\mathbf{S}^1/\mathbf{Z}_2$ (which describes the strong coupling limit of the $E_8 \otimes E_8$ heterotic string) approaches one of the fixed points?

The paper is organized as follows. Section (2) is a review of small instantons in the $SO(32)$ heterotic string. In section (3) we employ T-duality to argue the existence of a tensionless string in the low energy limit for a small $E_8 \otimes E_8$ instanton. In section (4) we derive a geometrical picture for the tensionless string from the M-theory. Section (5) discusses the relation to D-branes in type I and type IA. In the appendix we calculate the anomalies which are needed for the geometrical description of section (4).

2 Review of small instantons in the $SO(32)$ heterotic string

One of the ways in which string perturbation theory breaks down is by approaching singularities of the gauge bundle. This is what happens when the heterotic string is compactified down to 6D in such a way that the gauge bundle on the four compactified dimensions has an instanton whose size is such that the curvature at its core cannot be neglected. The limit of taking the size of the instanton to zero produces a soliton with a region in which the dilaton blows up. Those solutions were constructed in [7, 8, 9, 10].

In [1] the behavior of small instantons in the $SO(32)$ heterotic string was described. It was found that when a small instanton shrinks to zero size an extra gauge symmetry appears that is supported at the core of the instanton. In addition, hypermultiplets which transform in the $(\mathbf{2}, \mathbf{32})$ representation of $Sp(1) \otimes SO(32)$ as well as a singlet field appear in the massless spectrum as the instanton shrinks. When k instantons shrink at the same point the gauge group is $Sp(k)$ and the massless hypermultiplets transform in the representations $(\mathbf{2k}, \mathbf{32})$ of $Sp(k) \otimes SO(32)$ and the antisymmetric of $Sp(k)$ (which is reducible and decomposes to a singlet plus the irreducible $(\mathbf{2k^2} - \mathbf{k} - \mathbf{1})$).

3 Deductions from T-duality

The subject of our discussion is the zero size instanton in the heterotic $E_8 \times E_8$ string, but in order to be concrete let us think of it as compactified on K3, as in [1]. The coordinates $x_1, \ldots x_6$ will be the uncompactified dimensions on $\mathbf{R}^6$, x_1 denotes the time coordinate. The

six dimensional theory has a $(2,0)$ supersymmetry which is the minimum possible in 6D. (There are two left spinor generators and no right spinor generators. This reduces to $N=2$ in 4 dimensions which is really a $(2,2)$ supersymmetry.) Among other data the theory is defined with a left moving gauge bundle with instanton number 24. We will be interested in singularities in moduli space for which one or more instantons shrink to zero size. We assume that the gauge fields of the instanton are embedded in one of the two E_8-s. Taking the limit where the size of K3 is very large we can, as in [1], forget about the K3. The instanton can be thought of as being on $\mathbf{R}^4$.

Ten-dimensional $E_8 \otimes E_8$ and $SO(32)$ heterotic strings are distinct, but when compactified on a circle $\mathbf{S}^1$ of finite radius their moduli spaces are identical and both heterotic theories are in fact equivalent under T-duality [11].

It is therefore natural to ask how the $E_8 \otimes E_8$ heterotic theory compactified on K3, with an instanton that has shrunk, behaves upon further compactification on $\mathbf{S}^1$ down to 5D.

In what follows we would like to argue on the basis of T-duality that relates the $SO(32)$ heterotic string to $E_8 \otimes E_8$ heterotic string, that there exists a string-like object with zero-tension in the six-dimensional theory (that is the low-energy theory of an observer in the un-compactified dimensions). We recall that such a string-like object first appeared in type-IIB compactified on a K3 with a shrinking 2-cycle [4] and was called the "non-critical string" [3].

3.1 T-duality facts

Starting with an $E_8 \otimes E_8$ string on $\mathbf{R}^9 \times \mathbf{S}^1$ where the $\mathbf{S}^1$ is of radius r, we add the special Wilson loop on the ${\mathbf{S}^1}_r$ of the form (in the adjoint representation $\mathbf{248}$ of E_8):

$$W = \begin{pmatrix} \mathbf{I}_{120\times 120} & 0 \\ 0 & -\mathbf{I}_{128\times 128} \end{pmatrix} \tag{1}$$

This breaks $E_8 \otimes E_8$ down to $SO(16) \times SO(16)$. According to [11], the result is equivalent to an $SO(32)$ heterotic string on $\mathbf{R}^9 \times \mathbf{S}^1$ where $\mathbf{S}^1$ is of radius $\frac{1}{r}$ with the Wilson loop (in the fundamental representation $\mathbf{32}$ of $SO(32)$):

$$W = \begin{pmatrix} \mathbf{I}_{16\times 16} & 0 \\ 0 & -\mathbf{I}_{16\times 16} \end{pmatrix} \tag{2}$$

Which also breaks $SO(32)$ down to $SO(16) \times SO(16)$. This is the same Wilson loop that was used in [12, 13].

3.2 Application to small instantons

When we compactify one direction of $\mathbf{R}^6$ (in $\mathbf{R}^6 \times K3$) so that the six-dimensional small instanton wraps around the $\mathbf{S}^1$ of $\mathbf{R}^5 \times \mathbf{S}^1 \times K3$, massless states of the six-dimensional theory will become BPS saturated states of the five-dimensional theory on $\mathbf{R}^5$. $N = 2$ supersymmetry in 5D has one *real* central charge which is invariant under T-duality. The special Wilson loops W that were chosen above have the virtue of mapping the $SO(16) \times SO(16)$ symmetry group on one side (say heterotic on $E_8 \otimes E_8$) to the group on the other side (heterotic on $SO(32)$). So $SO(16) \times SO(16)$ quantum numbers of BPS states will not change under T-duality.

This is not the only good quality of W, and we will soon make use of its other properties as well.

We start with some massless state that exists in the "mysterious" six-dimensional theory of the $E_8 \otimes E_8$ small instanton. Compactifying on a radius $r \gg 1$ (with the special Wilson loop W) we expect to get a BPS state in the 5-dimensional theory which will become a BPS state with *the same* $SO(16) \times SO(16)$ quantum numbers in the $SO(32)$ theory compactified on a radius $\frac{1}{r}$. Conversely, if we knew what are the BPS states of the $SO(32)$ theory compactified to 5-dimensions on a radius $r' \ll 1$ we would expect to find massless states with the same $SO(16) \times SO(16)$ quantum numbers in the mysterious theory.

We know from [1] that on $\mathbf{R}^6$ the $SO(32)$ zero-size instanton has an additional $Sp(1)$ gauge symmetry and massless hyper-multiplets in the $(\mathbf{32}, \mathbf{2})$ representation of $SO(32) \times Sp(1)$. Thus, for $r' \gg 1$ the same theory on $\mathbf{R}^5 \times \mathbf{S}^1{}_{r'}$ has BPS states with the quantum numbers $(\mathbf{16}, \mathbf{2})$ (with the $\mathbf{16}$ corresponding to the $SO(16)$ in which the instanton was embedded). Note that from the explicit form (2) of W it follows that the other $(\mathbf{16}, \mathbf{2})$ fields which are in the fundamental representation of $SO(16)$ are multiplied by $-\mathbf{I}_{16\times 16}$ in (2). They thus have anti-periodic boundary conditions along the $\mathbf{S}^1$ and they give rise to massive states in 5D, which become very heavy once we decrease the radius of $\mathbf{S}^1$. [1]

[1]We are grateful to E. Witten for pointing this out as well as the special features of W in the next

What happens as we decrease r'? We know from [14, 15] that in the general case the vector-multiplet moduli space of the heterotic string on $K3 \times \mathbf{S}^1{}_{r'}$ has a "jump" singularity at $r' = 1$. BPS states can become unstable even at smooth points of the vector-multiplet moduli space (as in [16]). However, there will always be at least one BPS state (probably even with zero mass) in the $(\mathbf{16}, \mathbf{2})$ which has nothing to decay into. It will be stable. Actually, the special Wilson loop W that we used makes life easier. According to [12] the only Wilson line for which there is no extra enhanced gauge symmetry at some value of the radius is the Wilson line given by equation (2). We therefore expect no transitions as we take the limit of small radius.

We are led to the conclusion that the $E_8 \otimes E_8$ small instanton compactified on $\mathbf{S}^1$ contains massless states in the representation $\mathbf{16}$ of $SO(16)$. In six-dimensions, before the compactification on $\mathbf{S}^1$, we expect E_8 to be a gauge symmetry of the theory of the zero-size instanton. However, when we decompose the representations of E_8 under the subgroup $SO(16) \subset E_8$, the fundamental representation $\mathbf{16}$ of $SO(16)$ never appears!

Thus, the $\mathbf{16}$ states could not have been there *before the compactification*.

We find that there are massless states which appear when the theory is compactified on $\mathbf{S}^1$ with an arbitrarily big radius, but there are no corresponding massless states when the theory is not compactified.

Such a situation was, of course, encountered before. We remember it from type-IIB compactified on K3, discussed in [4]. This was the first appearance of the *tensionless non-critical string*. It was explained in [4] that a string with zero tension will produce such an effect. It is a question of "the order of the limits". A winding state of a string of tension ϵ on a circle of radius R will produce a state of mass of order ϵR. When we take $R \to \infty$ first, the state acquires infinite mass. When we take $\epsilon \to 0$ first, the state becomes massless.

In the next section we will see where this tensionless string "comes from" and explore what happens when we lift the tension ϵ.

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paragraph.

4 Strong coupling limit

Since the K3 is very large, we can replace it locally by $\mathbf{R}^4$ with a solitonic 5-brane charged under the dual of the (NS-NS field) $B_{\mu\nu}$ sitting at the origin of the $\mathbf{R}^4$ (and having all values of $x_1, \dots x_6$). Taking, in this situation, the asymptotic value of λ_{st} (i.e. at infinity of $\mathbf{R}^4$) to infinity doesn't change the nonperturbative phenomena that originate "from deep down the throat". This brings us to the M-theory compactified on the orbifold $\mathbf{S}^1/\mathbf{Z}_2$ with the radius of $\mathbf{S}^1$ becoming large [13]. The charge of the soliton corresponds to the charge of the "elementary" 5-brane of the M-theory [19]. We will assume that the small instanton becomes a 5-brane of the M-theory. According to [13] the gauge fields of the M-theory "live" only on the fixed points of $\mathbf{S}^1/\mathbf{Z}_2$ in such a way that each E_8 in $E_8 \otimes E_8$ comes from a different fixed point. Thus, in order for the 5-brane to describe massless states that are charged under one of the two E_8-s it must lie entirely on one fixed point. Since the distance between the two fixed points increases with the 10D coupling constant, and we are taking it to infinity, we end up with the M-theory on $\mathbf{R}^6 \times \mathbf{R}^4 \times \mathbf{R}^+$ with a 5-brane that sticks to the boundary at the position $\mathbf{R}^6 \times \vec{0} \times 0$ (the first $\vec{0}$ is the origin of $\mathbf{R}^4$ and the second is the fixed point of $\mathbf{R}^+ = \mathbf{R}^1/\mathbf{Z}_2$.)

How is the $(2,0)$ SUSY structure obtained? The 5-brane in 11D has $(4,0)$ SUSY on its world-volume (Indeed, it supports a multiplet comprising of a anti-self-dual two-form and 5 scalars). The extra $\mathbf{Z}_2$ imposed by the orbifold in the x^{11} direction leaves $(2,0)$ for the uncompactified six dimensions.

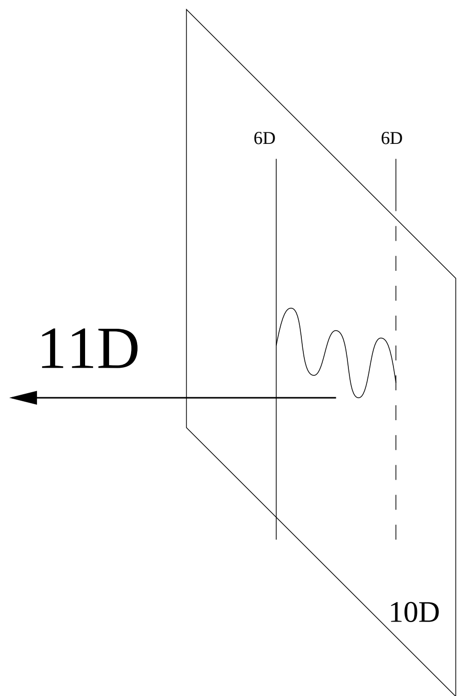

Fig. 1: A 5-brane close to the boundary of 11D and its image, with a 2-brane connecting them.

4.1 Appearance of a tensionless string

The setting is almost the same as in [5]. There we learn that a membrane can end on five-branes (see also the discussion in [17]). From [13] we also know that the membrane can end on one of the $\mathbf{Z}_2$ fixed points (which might be called a 9-brane).

Let us briefly review the construction of [5]. Consider a membrane stretched between two five-branes in eleven dimensional supergravity. The low energy dynamics of the five brane is described by an $N = 2$ $d = 6$ chiral tensor multiplet containing 5 scalars and an anti-self-dual antisymmetric tensor. We denote its field strength by T. The electric charge of the membrane with respect to the 3-form potential is given by integrating over an S^7 which surrounds the membrane

$$Q = \int_{S^7} *F,$$

F being the 4-form field strength. The boundary of the membrane is a string lying inside the five-brane world-volume. This string serves as a source for the anti-self-dual antisymmetric tensor

$$Q = -\int_{S_3} T.$$

When the five-branes are separated the low energy dynamics is given by 2 tensor multiplets and their moduli space is given locally by a symmetric space $SO(5,2)/(SO(5) \times SO(2))$. When the positions of the five branes coincide a tensionless string arises which carries no charge with respect to the $SO(2)$ group that acts on the two five-branes.

What happens in our case? We wish to identify the zero size instanton in the heterotic theory with an M-theory configuration which is the limit of a 5-brane approaching the boundary (the $\mathbf{Z}_2$-fixed point which we will later also refer to as the 9-brane) until it becomes submerged in it. Implementing what we studied in [5] and [13] we deduce that two kinds of "new" massless excitations can appear. Those are a 2-brane stretched between the 5-brane and its $\mathbf{Z}_2$ mirror image and a 2-brane that starts on the 5-brane and end on the 9-brane. In the limit when the distance from the 5-brane to the 9-brane is zero, both excitations will be observed as 1-branes of zero tension in the uncompactified 6D.

In the rest of the paper we will support the above argument with some further tests:

- The instanton tension scales like $\frac{1}{\lambda^2}$ and the five-brane has the same behavior (does not depend on the radius) after a Weyl rescaling.
- We will discuss the appearance of the hypermultiplet in the **16** of $SO(16)$ after compactification on $\mathbf{S}^1$ and breaking of $E_8 \otimes E_8$ down to $SO(16) \times SO(16)$.
- After the $\mathbf{S}^1$ compactification, it is possible to transform to the type-I string as in [13]. We will identify the counterpart of the proposal in the type-I language.
- Collapse of k small instantons at the same point will be discussed as well as its relation to type I and small $SO(32)$ instantons.
- Separation of the 5-brane from the 9-brane will be identified as an $Sp(1)$ Wilson loop in the T-dual $SO(32)$ theory (again after further compactification).

4.2 low-energy fields

In the previous section we described the low energy spectrum of a membrane stretched between two five-branes ($5-5$ membranes). We now turn to describe the low energy fields of $5-9$ membranes. So we consider a membrane stretched between a five-brane and a nine-brane as in figure 2. Note that a small instanton differs from this configuration by having the five-brane located at the nine-brane. We nevertheless, for the propose of describing the low energy fields, keep the branes apart.

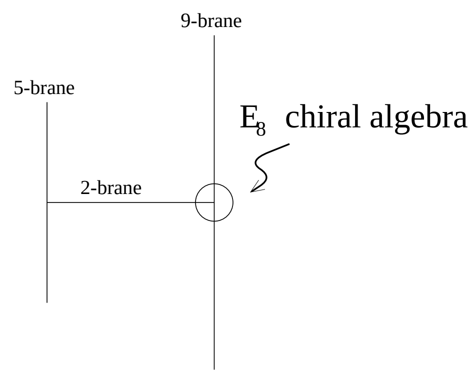

Fig.2: A chiral E_8 that appears on the intersection of a 2-brane with a fixed point

Let us analyze the configuration of Fig.2, where we have the M-theory on $\mathbf{R}^1/\mathbf{Z}_2$ (that is $\mathbf{S}^1/\mathbf{Z}_2$ near the $x^{11}=0$ fixed point, with the other fixed point very far away so that we can forget about it). We also have a (cosmic) 5-brane at distance r from the fixed point (i.e. at $x^{11}=r$ and say $x^7=\cdots=x^{10}=0$). We assume that $r \gg \alpha'^{1/2}$ and that there is a cosmic 2-brane stretched between the 5-brane and the fixed point, that is at position

$$x^3 = x^4 = \cdots = x^{10} = 0, \qquad 0 \leq x^{11} \leq r \tag{3}$$

It is now legitimate to ask how an eleven-dimensional observer at a scale that is much larger than α'^5 describes the low-energy physics. The low-energy field theory comprises of various fields. First there are the 11D supergravity fields that live in the 11D bulk. Then there are 10D fields that live just on the ten dimensional hyper-surface $x^{11}=0$ and interact

with the boundary value of the 11D fields on $x^{11} = 0$.

The physics at the $x^{11} = 0$ end was worked out in [13] on the basis of anomaly cancelations both in 11D and on the 2-brane. It was argued there that on the fixed point there is an E_8 10D Super Yang-Mills theory, and on the 2D intersection there is a chiral level one E_8 current algebra. On the 2-brane bulk there lives the dimensional reduction of super Yang-Mills from 10D down to 3D (see [18]). In 3D we get 8 scalars (using the fact that a vector is dual to a scalar in 3D) that represent transverse oscillations. Now we come to the question of what lives on the 5-brane. Generically, there lives a tensor multiplet and a hyper-multiplet – together we get one antisymmetric tensor field B^{+}_{mn} (with an anti-self-dual field strength) and 5 scalars that represent transverse oscillations of the 5-brane. However, when the 5-brane is stuck on the boundary (9-brane), the situation changes a little bit. Out of the 5 scalars, the one that corresponds to a translation in the x^{11}-th direction, i.e. away from the boundary, is no longer a modulus. When we decompose the $(4, 0)$ SUSY multiplet of the $B^{(+)}_{\mu\nu}$ and 5 scalars under the surviving $(2, 0)$ SUSY we find two multiplets. One comprises of $B^{(+)}_{\mu\nu}$ and one scalar, and the other is the usual hyper-multiplet of 4 scalars. Since the modulus of the first one corresponds to x^{11} translations and is frozen as we argued above, we do not see the tensor multiplet in the resulting 6D theory (see also the comment in [19]). The exact mechanism by which the tensor multiplet becomes "frozen" is not clear to us, but it must involve tensionless string dynamics, since as we will see below, after compactification to 5D on an arbitrarily big radius the transverse directions re-appear and correspond to $Sp(1)$ Wilson loops along the 6th compactified direction. In fact in section (5.2) we will conjecture that in six dimensions the small instanton corresponds to a point in the moduli space that connects two different phases. One phase is in which the instanton has nonzero size. The other phase was discussed in [22] where it was pointed out that the anomaly can be cancelled by a gauge bundle with instanton number 23 (i.e. one unit less) and a five brane located in an arbitrary position in the x^{11} direction.

The remaining question is what lives on the intersection of the 5-brane with the 2-brane. We show in the appendix that anomaly cancelation requires an anomaly free 2D theory. So to summarize we have the following fields

1. 11D supergravity in the bulk

2. Super E_8 Yang-Mills in 10D on the fixed point
3. A tensor multiplet and a hyper-multiplet in 6D on the 5-brane bulk.
4. Dimensional reduction of $U(1)$ super-YM down to 3D on the 2-brane bulk
5. A chiral E_8 level one current algebra on the 2D boundary of the 2-brane on the fixed point.
6. We assume that there is nothing on the boundary where the 2-brane ends on the 5-brane.

4.3 Recovery of the fundamental representation

We argued previously that the effective theory in 6D is governed by a tensionless string, whereas compactification on $\mathbf{R}^5 \times \mathbf{S}^1$, where the $\mathbf{S}^1$ is arbitrarily large, yields an effective *field* theory in 5D. When we compactified on $\mathbf{S}^1$ with the special Wilson loop W of (1), we obtained the theory of [1] and in particular hypermultiplets in the **16** of $SO(16)$ appeared. Since we know from the previous subsection what are the various low-energy fields, let us try to recover the **16** states. So we compactify the picture of the previous section on $\mathbf{S}^1$ with the special Wilson loop W. In the previous picture the 5-brane was separated from the fixed-point. We will argue in section (5) that in the $SO(32)$ picture of [1] the separation corresponds to an $Sp(1)$ Wilson loop along $\mathbf{S}^1$. We will see there that the $(\mathbf{2}, \mathbf{16})$ states that we are looking for have a mass which is proportional to the Wilson loop and to the separation in the M-theory picture. They are still in a reduced representation, and so are BPS states. The BPS argument allows us to work with the separated 5-brane and 9-brane. BPS excitations of D-branes correspond to the *vacuum* of the low-energy that lives *on the D-brane*. Thus, we have to calculate the vacuum degeneracy of the low-energy theory of the previous subsection.

We realize the E_8 chiral algebra that lives on the 2D boundary at the fixed point by 16 chiral fermions (with both NS and R boundary conditions). We recall that the E_8 ground states are

$$\psi^i_{-\frac{1}{2}}\psi^j_{-\frac{1}{2}}|0\rangle \tag{4}$$

(120 states in the NS sector), plus the 128 states $|\alpha\rangle$ in the R sector. Because of a normal

ordering constant all the 248 states have zero energy. After the Wilson loop only the $\psi^i_{-\frac{1}{2}}|0\rangle$ states are the surviving low-energy excitations. This is because:

1. The Wilson loop adds a normal-ordering contribution to L_0 which changes the $\psi^i_{-\frac{1}{2}}|0\rangle$ from tachyonic to massless.
2. The GSO projection projected out the $\psi^i_{-\frac{1}{2}}|0\rangle$ and left the $|0\rangle$ before the addition of the Wilson loop. The operators ψ^i are invariant under W, but since W acts as (-1) on the states in the R sector, it means that the GSO charge operator $(-)^F$ has an extra $(-)$ sign when acting on the vacuum $|0\rangle$. The result is that $\psi^i_{-\frac{1}{2}}|0\rangle$ are not projected out while the tachyonic $|0\rangle$ is projected out (see [20] for a related discussion).

The $\mathbf{2}$ in the $(\mathbf{2},\mathbf{16})$ just corresponds to wrapping the 2-brane in opposite directions around the $\mathbf{S}^1$.

5 Relation to type-I

What is the physical meaning, for the heterotic string, to endowing the tensionless string with a small tension of $\epsilon > 0$? We need to know what happens to the heterotic string when we separate the 11D 5-brane from the fixed point (of $\mathbf{S}^1/\mathbf{Z}_2$). We begin by asking ourselves how the Chan-Paton factors of two, discovered in [1] manifest themselves in E_8 instantons in the M-theory.

To relate E_8 theory instantons to type-I Dirichlet 5-branes we will follow exactly the same steps as in [13]. First we start with M-theory on $\mathbf{S}^1/\mathbf{Z}_2$. As in [13], it must be mentioned that the $\mathbf{Z}_2$ doesn't act *only* on the $\mathbf{S}^1$ but is a *combined* action of a reflection of the $\mathbf{S}^1$ and

$$A_3 \longrightarrow -A_3 \tag{5}$$

where A_3 is the 3-form potential of 11D supergravity. This transformation means that the $\mathbf{Z}_2$ reverses the world-volume orientation of the fundamental 2-brane in 11D (to which A_3 couples). The orientation of the 5-brane is invariant because it couples to the dual to A_3 (which is invariant because of an 11D ϵ-symbol in the formula). The above transformation of A_3 is necessary in order to find a surviving 2-form in 10D which would couple to the fundamental heterotic string [13].

As explained in [13], after compactification on another $\mathbf{S}^1$ down to 9D, we find a theory that is an orbifold of type-IIA on $\mathbf{S}^1/\mathbf{Z}_2$, but the $\mathbf{Z}_2$ acts both on the $\mathbf{S}^1$ and exchanges the orientation of the type-IIA worldsheet (because the type-IIA string is the wrapped 2-brane of the M-theory). This theory was called the type-IA. T-duality (which inverts the radius of the $\mathbf{S}^1/\mathbf{Z}_2$) brings this theory to the usual type-I. Now, insert a 5-brane in type-I. According to [1] this 5-brane has a Chan-Paton factor that can have two values. There are world-volume $Sp(1)$ gauge fields living on the type-I 5-brane. The D-brane description [21] of the 5-brane assigns Neumann boundary conditions in the $\mathbf{S}^1$ direction (on which the type-I 5-brane is wrapped). T-dualizing this configuration we go to the type-IA, in which there are orientifold planes at each fixed point $x^{11} = 0, \pi$. The x^{11} coordinate in type-IA corresponds to an $SO(32)$ Wilson line eigenvalue, in the sense that turning on an $SO(32)$ Wilson line in type-I will cause a translation in the position of the 32 8-branes in the x^{11} direction in type-IA. The type-I 5-brane which was wrapped on the x^2 direction becomes two 4-branes positioned symmetrically with respect to Z_2 in type-IA at specific x^{11} coordinates. These coordinates corresponds to the value of an $Sp(1)$ Wilson loop around the x^2 direction in type-I. We note that the $Sp(1)$ Wilson loop corresponds to a different phase in the moduli space, and the small instanton can be excited into this phase as well. In 6D there are only the excitations by VEVs to the hyper-multiplets in the fundamental representation $(\mathbf{2}, \mathbf{32})$ which describes the instanton (in the heterotic $SO(32)$) acquiring a finite size [1]. Upon compactification to 5D, the vector-multiplet, which had no scalars in 6D, has the Wilson loop value as its scalar super-partner. When this Wilson loop is turned on at a generic value, all the hyper-multiplets become massive and thus have VEV zero. This means that we are on a different phase.

Returning to type-IA, we find that an $Sp(1)$ Wilson loop (in type I) splits the 4-brane (which was T-dual to the original 5-brane) into two separate mirrored 4-branes. The type IA came from an orbifold of type-IIA , the latter being the M-theory wrapped on the 11th direction. Thus we end up with two 5-branes in the M-theory at two different coordinates in the x^{11} direction (which is the direction of the $\mathbf{S}^1/\mathbf{Z}_2$). These two coordinates must be mirror images of one another under the $\mathbf{Z}_2$. We thus recovered the M-theory picture, but we have also learned that separating the two 5-branes in the M-theory (away from the orbifold fixed point where the E_8 lives), corresponds not to an instanton of finite size, but to a different

phase. We elaborate on the phase transition in section (5.2).

We can now substantiate the claim made before that the $(\mathbf{2},\mathbf{16})$ correspond to winding states of the non-critical string. When a small $Sp(1)$ Wilson loop of magnitude proportional to ϵ is turned on, the hypermultiplets $(\mathbf{2},\mathbf{16})$ become massive with a mass proportional to ϵ. On the other hand, we saw that in the M-theory the 5-branes are now separated a distance proportional to ϵ so the non-critical string connecting the 5-branes as in [5] as well as the non-critical string connecting the five-brane to the $\mathbf{Z}_2$ fixed point have now a tension proportional to ϵ in correspondence with the expected mass of their winding states.

5.1 k instantons

The above picture leads us to conclude that when an $E_8 \otimes E_8$ instanton goes to zero size, a five brane emerges on one of the Z_2 fixed points corresponding to one of the E_8 gauge groups in which the instanton is embedded. The analysis of the massless spectrum goes as follows. We first consider the case in which the five-brane is separated from the nine-brane. This corresponds, as we argued, to a $Sp(1)$ Wilson line after compactification on a circle. In this case the massless spectrum contains one tensor multiplet on the six dimensional world volume of the five brane. The other case is when the five-brane is inside the nine-brane. This corresponds to a small instanton. We have no tensor multiplet but instead a tensionless $5-9$ string emerges on the six dimensional worldvolume from a membrane connecting the five-brane and the nine-brane. In addition there is a $5-5$ non-critical tensionless string. This string emerges from the intersection of a membrane which connects the five brane with its own Z_2 image. In both cases the string tension is proportional to the distance between the branes.

This can be generalized to a configuration for which k instantons shrink to zero size at the same point. In the dual picture we have four types of tensionless strings. One which, as before, connects the 5-brane to the 9-brane. There are k such strings which transform in the fundamental representation of $SO(k)$. The second type is a string which connects two distinct 5-branes. There are $k(k-1)/2$ such strings and they transform in the adjoint representation of $SO(k)$. The third type is a string which connects a five-brane to a Z_2 image of another distinct five brane. There are also $k(k-1)/2$ such strings and they too

transform in the adjoint representation of $SO(k)$. The fourth type is a string which emerges from a membrane which connects a five-brane with its image. There are k such strings and they transform in the fundamental representation of $SO(k)$.

In addition if we separate the five-branes from the nine-brane (we can only give it a meaning after compactification on a circle and, as discussed before, giving a Wilson line to the $Sp(k)$ group) the $5-9$ strings and the $5-5$ strings which connect two Z_2 images get nonzero tension while k massless tensor multiplets on the world volume of the five-branes emerge. The tensors transform in the fundamental representation of $SO(k)$. There are still $k(k-1)/2$ tensionless $5-5$ strings connecting the distinct five-branes.

When we consider the reduction of this configuration to type I by a double zero radius limit, namely taking both x^2, x^{11} to be very small, we should recover the dual picture for small instantons on the $SO(32)$ heterotic string. We recall that when k instantons shrink to zero size in $SO(32)$ a nonperturbative $Sp(k)$ gauge group appears with hypermultiplets in the antisymmetric representation of $Sp(k)$ and also hypermultiplets in the $(\mathbf{2k}, \mathbf{32})$ representation of $Sp(k) \times SO(32)$. We see that if a $5-5$ $(5-9)$ tensionless string gives rise to two (one) six dimensional hypermultiplets and two (one) six dimensional vector fields after compactification on $S^1 \times S^1/Z_2$, then we reproduce the right spectrum for the gauge group $Sp(k)$ and its antisymmetric representation. The counting gives $k+2\cdot[2\cdot k(k-1)/2+k] = k(2k+1)$ vector multiplets in the adjoint of $Sp(k)$, and $k + 2 \cdot [2 \cdot k(k-1)/2] = k(2k-1)$ in the antisymmetric representation of $Sp(k)$. The k tensionless strings which connect the 5 branes with the 9 brane will give rise to the remaining hypermultiplets.

5.2 Phase transitions via tensionless strings

In four dimensions, different phases can be connected at points where extra massless BPS particles appear [16]. The two phases differ by the number of hypermultiplets and the number of vector multiplets. In six dimensions there is no central charge for particles (this is related to the fact that vector multiplets have no scalar components) and so such a mechanism can not bring about phase transitions. However, there do exist central charges for 1-branes in 6D. The BPS formula determines the tension of BPS 1-branes in terms of expectation values of scalar components of tensor multiplets. Can tensionless 1-branes connect two different

phases?

In fact the tensionless string that we have discussed in the previous sections is a transition point between two phases. We can deform the small instanton in two ways. One way is to make the instanton size nonzero. This deformation is parameterized by 29 hypermultiplets as can verified by the dimension of the moduli space of an E_8 instanton [22]. The other way is to separate the five-brane from the nine-brane thus giving the non-critical string a tension. This phase has one extra tensor multiplet whose scalar component corresponds to the x^{11} distance of the five-brane to the nine-brane. This phase was discovered in [22] as a new way of cancelling the anomaly equation. Passing from one phase to the other through the tensionless string we trade 29 hypers with 1 tensor as the gravitational anomaly in six dimensions asserts.

Upon compactification to 5 dimensions the tensor multiplet becomes a vector multiplet, tensionless strings become massless particles and the phase transition mechanism becomes an analog of the four dimensional phase transition.

6 Discussion

One of the by-products of the recent developments in nonperturbative string theory is the discovery of a mysterious six-dimensional low-energy theory which is not a field theory [4]. This theory contains tensionless strings, but it is not clear how to quantize them, neither is it clear whether the theory can be described by quantizing tensionless strings. There have been some suggestions for theories of tensionless quantized strings [23]. [2] Recently, tensionless string theories have been constructed directly from the M-theory [5], and indeed we have used the ideas of the latter construction.

We have argued on the basis of T-duality that the small instanton of the $E_8 \otimes E_8$ heterotic string might be described by a theory which contains tensionless strings in 6D. It seems that the tensionless strings arising from $E_8 \otimes E_8$ small instantons are novel in that they can couple to a six-dimensional gauge field. (By taking k 5-branes in [5] we obtain an $Sp(k)$ gauge group only *after compactification* to 5D. The would-be gluons are 1-branes in 6D.)

[2] We are grateful to David Gross for bringing these references to our knowledge.

The 11 dimensional picture that we "drew" might be interpreted as a bound-state of a 5-brane and a 9-brane. The mechanism for this binding is related to tensionless string dynamics and it might be interesting to explore it further.

It would be interesting to study the dynamics of the various low energy excitations which emerge on the intersection of the five-branes and nine-branes thus gaining more understanding on the strong coupling phase of small $E_8 \otimes E_8$ instantons and the transition to the weak coupling phase. It is plausible that the transition includes physics of tensionless strings. One way of studying these strings is through a compactification to lower dimensions.

One more point is in order. When the instantons are embedded symmetrically in $E_8 \times E_8$, it was shown in [22] that there exists a dual description of the $\mathbf{R}^6$ physics in terms of a weakly coupled heterotic string. The non-perturbative $SU(2)$ which appears for a zero-size instanton is mapped, in the dual description, to a perturbative enhanced gauge symmetry that appears for special points in the Narain moduli space. This picture appears to be very different than what we have argued in the previous sections. In particular, we argued that the 6D low-energy theory is a non-critical string and this seems very remote from a field theory. The reason for the discrepancy is that in this paper we focussed only on the contribution of the small instantons to the 6D low energy spectrum. In order to obtain the *complete* 6D low energy spectrum one needs to analyze the coupled system of the tensionless strings with the rest of the fields (coming from the bulk of the K3 where the interaction is through the E_8 gauge fields). This problem involves tensionless string dynamics, and it is plausible that as a result of that only the particle-like excitations of the string remain. It would be very interesting to understand under what circumstances that happens.

We have seen a new type of phase transitions in six dimensions by the appearance of a tensionless string. In particular we have connected the phase of a finite size instanton with the phase of a five-brane in the eleven dimensional bulk in this way. The appearance of a tensionless string seems to be important for understanding other cases of phase transitions in six dimensions. In particular we recall another situation in which a tensionless string seems to be relevant. As was explained in [22, 19], when the gauge bundle is embedded asymmetrically in $E_8 \otimes E_8$, there is a phase transition at strong coupling. The description of this phase transition in terms of tensionless strings as well as the role of tensionless strings

in other phase transitions is under current investigation [24].

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Appendix: Gravitational anomalies on the 2-brane

In section (4.2) we assumed that nothing special "lives" on the 2D intersection of the 5-brane and the 2-brane. This was based on the fact that there are no gravitational anomalies supported at that end.

Let's briefly recall how the anomalies were calculated in [13]. They had a 2-brane stretched between the two fixed points and they wanted to determine what lives on the two 2D boundaries from a knowledge of the behavior of the fields on the 2-brane bulk. The fields on the 2-brane bulk are the reduction of $U(1)$ super YM in 10D down to 3D (see [18]). In 3D we get 8 scalars (using the fact that a vector is dual to a scalar in 3D) that represent transverse oscillations together with their super-partners that are spinors in the $(\mathbf{2}, \mathbf{8}'')$ of $SO(1,2) \times SO(8)$ (where the notation of [13] was that $\mathbf{8}, \mathbf{8}'$ and $\mathbf{8}''$ were the vector and two spinors of $SO(8)$ and $\mathbf{2}$ was the spinor of $SO(1,2)$. Those are the spinors that obey $\Gamma_1\Gamma_2\Gamma_{11} = 1$ which is the BPS condition of the 2-brane. Since the boundary conditions that the bulk fields satisfy is known, the contribution of the bulk fields to the gravitational anomalies under a reparametrization of the x^1, x^2, x^{11} coordinates can, in principle, be determined. In [13] a shortcut was made by arguing that the anomaly is supported just at the ends so that it is sufficient to consider reparametrization of the x^1, x^2 plane that is independent of the x^{11} coordinate, and the anomaly of the latter is determined by the zero modes (in the x^{11}-direction) of the fields. It was also explained that the $\mathbf{8}''_-$ fermions produce the same gravitational anomalies as right-moving RNS fermions and superconformal ghosts, because

the $SO(8)$ degrees of freedom are not really independent of the world-sheet, since the $SO(8)$ is the structure group of the normal bundle of the world-sheet in 10D.

After this review of [13], we wish to repeat their calculation for the case at hand of a 2-brane stretched between a 5-brane and a $\mathbf{Z}_2$ fixed point. It is intuitively clear that since the 5-brane has no $\mathbf{Z}_2$ there is no 2D chirality condition from that end, and we should not get any anomaly, so that the total 2D anomaly should be half that of a 2-brane stretched between two $\mathbf{Z}_2$ fixed points. Let's see how it happens. At the fixed point, the boundary condition on the fermions reads:

$$\Gamma_{11}\psi(x^{11}=0)=\psi(x^{11}=0) \tag{6}$$

Since the bulk fermions satisfy the BPS condition

$$\Gamma_1\Gamma_2\Gamma_{11}\psi=\psi \tag{7}$$

(this determines what we mean by the $\mathbf{2}$ of $SO(1,2)$ above.) On the 5-brane end there is no $\mathbf{Z}_2$ projection condition, but there is the condition

$$\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\psi(x^{11}=r)=\psi(x^{11}=r) \tag{8}$$

This condition comes from the fact that the bosons of the 2-brane that describe oscillations in the directions transverse to the 5-brane have Dirichlet boundary conditions at the 2D boundary of the 2-brane. Supersymmetry relates these boundary conditions to the fermions, but we have to remember that at the 2D boundary only half the supersymmetry survives (compared with the 2-brane bulk) because the 5-brane has its own BPS condition. To find the massless fermions that a 2D observer sees we must take the fields to be independent of x^{11}. So, we end up with those fields in $(\mathbf{2},\mathbf{8}'')$ of $SO(1,2)\times SO(8)$ that satisfy the additional conditions (which break $SO(1,2)\times SO(8)$):

$$\Gamma_{11}\psi=\Gamma_1\Gamma_2\Gamma_{11}\psi=\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\psi=\psi \tag{9}$$

as well as being in $\mathbf{8}''_-$.:

$$\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}\psi=-\psi \tag{10}$$

Let's decompose those fermions under $SO(1,1)\times SO(4)\times SO(4)$. The surviving spinors are

$$(\mathbf{2},\mathbf{2}')_- \tag{11}$$

The final step is as in the last paragraph of chapter (2) of [13]. Since the second $SO(4)$ (in the decomposition $SO(8) \supset SO(4) \times SO(4)$) decouples because it is in the directions transverse to the 5-brane that is assumed fixed, we see that we need two times the anomaly of fermions in the $\mathbf{2}_-$ where the $\mathbf{2}$ is the negative chirality spinor of the normal bundle N_4 of the 2D surface Σ in the 5-brane bulk M_6 (i.e. $TM_6 = N_4 \oplus T\Sigma_2$ where subscripts denote dimensions). The index for the gravitational anomalies for a chiral spinor in the representation $\mathbf{r}$ is

$$\hat{I}_{\frac{1}{2}}(F,R) = \mathrm{tr}_{\mathbf{r}}\{e^{iF}\}(1 + \frac{1}{48}\mathrm{tr}\{R^2\} + \cdots). \tag{12}$$

In 2D the 4-form part of the above formula equals

$$\frac{c-\bar{c}}{24}\mathrm{tr}\{R^2\}, \tag{13}$$

where c and $\bar{c}$ are the left and right central charges respectively. In our case $\mathbf{r} = \mathbf{2}$ and we also know that in the fundamental representation of $SO(4)$ the bundle $N \oplus T\Sigma$ is trivial so that

$$\mathrm{tr}_{\mathbf{4}}\{F \wedge F\} + \mathrm{tr}\{R \wedge R\} = 0. \tag{14}$$

Now we find

$$\begin{aligned} \mathrm{tr}_{\mathbf{2}}\{1\} &= 2, \\ \mathrm{tr}_{\mathbf{2}}\{F \wedge F\} &= \frac{1}{4}\mathrm{tr}_{\mathbf{4}}\{F \wedge F\} = -\frac{1}{4}\mathrm{tr}\{R \wedge R\}. \end{aligned}$$

So we find

$$\hat{I}_{\frac{1}{2}}(F,R) = (\frac{2}{48} - \frac{1}{2}\cdot(-\frac{1}{4}))\mathrm{tr}\{R^2\} = \frac{1}{6}\mathrm{tr}\{R^2\}. \tag{15}$$

For comparison, the $\mathbf{1}$ representation would give

$$\frac{1}{48}\mathrm{tr}\{R^2\}. \tag{16}$$

So the contribution of $\mathbf{2}_-$ to the anomaly is like 8 left moving fermions i.e. $c - \bar{c} = 4$. Altogether the anomaly of 11 is twice as much, that is 8. (For a check let's see how the $\mathbf{8}''_-$ of [13] gives $c - \bar{c} = 16$. We find

$$\begin{aligned} \mathrm{tr}_{\mathbf{8}''}\{1\} &= 8 \\ \mathrm{tr}_{\mathbf{8}''}\{F \wedge F\} &= \mathrm{tr}_{\mathbf{8}}\{F \wedge F\} = -\mathrm{tr}\{R \wedge R\} \\ \hat{I}_{\frac{1}{2}}(F,R) = (\frac{8}{48} &- \frac{1}{2}\cdot(-1))\mathrm{tr}\{R^2\} = \frac{16}{24}\mathrm{tr}\{R^2\} \end{aligned} \tag{17}$$

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